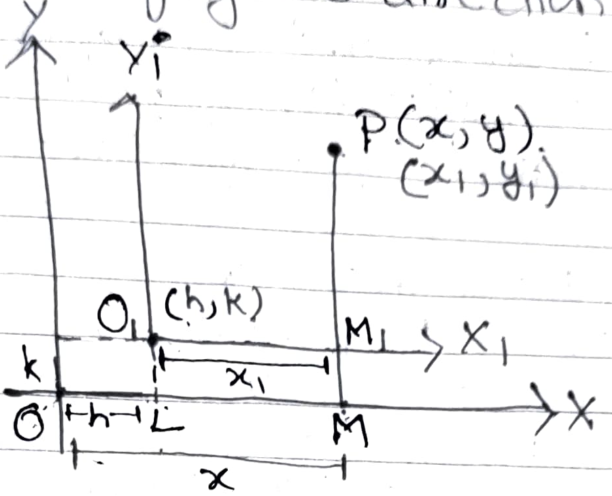
Co-ordinate Geometry

Transformation of Axes

Change of Origin

i.e, to change the origin of co-ordinates without changing the direction of axes



$PM = y$
$PM_1 = y_1$

Let OX and OY be the orignal axes and O_1X_1 and O_1Y_1 be the nex axes parallet to orignal.

Let O_1 be the new origin and (h, k) be its co-ordinates reffered to OX and OY.

$O L = h$, $O_1 L = k$

Let P be any point on the plane and its co-ordinates be (x, y) reffered to OX & OY and co-ordinates (x_1, y_1) reffered to O_1X_1 & O_1Y_1

Draw $PM \perp OX$

$\Rightarrow \quad x = OM = OL + LM$

$= OL + O_1M_1 = h + x_1$

$x = h + x_1$

$y = PM = MM_1 + PM_1 = O_1L + PM_1$

$= k + y_1 \Rightarrow y = k + y_1$

So, if the origin is transfered to a point (h, k) then the co-ordinates of the Point P changes from (x, y) to (x_1+h) and (y_1+k) respectively.

Ques Transform to parallel axes through the point $(1, -2)$, ~~those~~ for the equation $2x^2 + y^2 - 4x + 4y + 3 = 0$

Soln Here $h = 1$; $k = -2$

$\therefore$ We substitute $x = x+1$ and $y = y-2$

So the required equation is

$2(x+1)^2 + (y-2)^2 - 4(x+1) + 4(y-2) + 3 = 0$

$\Rightarrow 2(x^2+2x+1) + (y^2-4y+4) - 4x - 4 + 4y - 8 + 3 = 0$

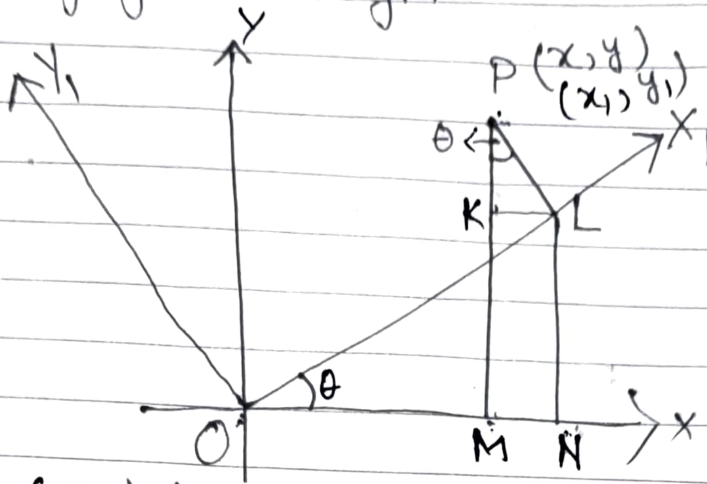
$\Rightarrow 2x^2 + \cancel{4x} + 2 + y^2 - \cancel{4y} + \cancel{4} - \cancel{4x} - \cancel{4} + \cancel{4y} \overset{-5}{\cancel{-8}} = 0$

$\Rightarrow \boxed{2x^2 + y^2 = 3}$

Ans

Change of Direction of axes

i.e., changing direction of axes without Changing the origin



Let (x, y) be the coordinates of Point P reffered to the orignal axes OX and OY.
Let (x_1, y_1) be the co-ordinates of P reffered to the new axes OX_1 and OY_1.

Let $\angle XOX_1 = \theta$

Draw $PM \perp OX$ and $PL \perp OX_1$

Also draw $LN \perp ON$ and $LK \perp PM$

Now.,

$$x = \cancel{PM}\ OM = ON - MN$$
$$= OL\cos\theta - PL\sin\theta$$ Old axes
$$= x_1\cos\theta - y_1\sin\theta$$
$$y = PM = PK + KM$$
$$= PL\cos\theta + OL\sin\theta$$
$$= y_1\cos\theta + x_1\sin\theta$$

∴ We get

(New Axes) $x_1 = x\cos\theta + y\sin\theta$ & $y_1 = -x\sin\theta + y\cos\theta$

	x	y
x_1	$\cos\theta$	$\sin\theta$
y_1	$-\sin\theta$	$\cos\theta$